GROUP-BASED FAILURE EFFECTS ANALYSIS

K. JENAB and B. S. DHILLON

University of Ottawa, Mechanical Engineering Department
P.O. Box 450, Stn. A, Ottawa
Ontario, K1N 6N5, Canada
Dhillon@genie.uottawa.ca

Received 21 June 2004
Revised 01 November 2004

This paper presents the multi-based experts Failure Effects Analysis (FEA). The experts’ opinions differ substantially because the experts do not often agree on the level of the failure factors (failure probability, non-detection probability, severity of effect, and expected cost) and the functions/subsystems attributes (e.g., importance). Therefore, conflict always occurs in Group-based Failure Effects Analysis (GFEA). The approach uses fuzzy Risk Priority Category (RPC) and group decision-making techniques to study both the failure effects on the functions/subsystems and the failure risk category with uncertain information. In addition, the approach uses the compensated operators to allow the tradeoffs either among failure factors or among functions/subsystems attributes. A solved example is presented to demonstrate the Group-based Failure Effects Analysis (GFEA) application.

Keywords: Failure Mode and Effects Analysis (FMEA); failure analysis; risk analysis.

1. Introduction

Failure Effects Analysis (FEA) is a fundamental risk analysis process involving information acquisition, modeling, analysis, and decision, which result in the physical design improvement. In recent years, the published literature pertaining to FEA has been concerned with developing either deterministic or fuzzy models in order to identify, prioritize and eliminate the potential failures in the system. Moreover, some approaches focus on application of decision-making techniques leading to improve the reliability, quality, and safety.

In deterministic models, the Risk Priority Number (RPN) and Pareto Chart (PC) were used as the principal knowledge acquisition to represent and score the
system failure effects. The RPN is multiplication of failure factors (occurrence probability, non-detection probability, severity of impact, expected cost). These factor values range between ‘1’ and ‘10’. In Ref. 5, to bring design of Failure Mode and Effects Analysis (FMEA) in line with accepted probabilistic risk management theory, the Society of Automotive Engineers (SAE) has replaced RPN metric with the Critically Matrix (CM). The CM shows severity of a failure against its probability of occurrence. However, the use of CM alone cannot compensate the major weakness of the SAE approaches and of others. Thus, to meet the SAE requirements and provide a robust FMEA model, the Bayesian Network was proposed and called BN-FMEA.

However, due to the shortcomings of deterministic RPN-based approaches (e.g., BN-FMEA) as pointed out in Refs. 4, 6 and 7, the RPN has been replaced with Risk Priority Category (RPC) for performing fuzzy FEA in the system. Using decision support system, the earliest fuzzy FEA methods can be classified as shown in Table 1.

In Refs. 6 and 9, the fuzzy method based on linguistic variables, Grey theory, and Maximin method was used to determine an RPC to evaluate the risk level of the system. The approaches allow considerable weighting of severity factor associated to a cause of failure. Due to using both linguistic variables for evaluating failure factors and Grey theory to prioritize the risk of failure without utility function, the proposed methods can be considered as a breakthrough in FEA. However, one drawback of these approaches is that the weight of criteria and the scores of the failure causes must be assigned subjectively. Later, in Refs. 10 and 11, the Analytic Hierarchy Process (AHP) technique was proposed to find the preferential weight of the failures in order to determine the failure risk priority number. This approach based upon three principles: decomposition, comparative judgements, and the synthesis of priorities. It has several shortcomings for failure effects analysis, such as man-made inconsistency in pair wise comparisons, and rank reversal when new failures are introduced. Thus, the knowledge base rule as a new approach for FEA

<table>
<thead>
<tr>
<th>Method category</th>
<th>Method name</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximin Methods</td>
<td>Chang et al. approach</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Puente et al. approach</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Xu et al. approach</td>
<td>14</td>
</tr>
<tr>
<td>Linguistic Methods</td>
<td>Kwok et al. approach</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Pillary et al. approach</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Wang et al. approach</td>
<td>17</td>
</tr>
<tr>
<td>AHP Methods</td>
<td>Bozdag et al. approach</td>
<td>11</td>
</tr>
<tr>
<td>TOPSIS Methods</td>
<td>Braglia et al. approach</td>
<td>16</td>
</tr>
<tr>
<td>Weighting Methods</td>
<td>Umano et al. approach</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Wang et al. approach</td>
<td>20</td>
</tr>
</tbody>
</table>
was introduced. In Ref. 12, using linguistic variables, authors developed the fuzzy single-based expert rules to determine the RPC of the failure. The approach utilizes Grey theory to avoid the use of utility function. However, FEA is a system technical risk analysis that extracts knowledge about the potential failure mode from a group of experts so-called expert knowledge base system with possible conflict in experts’ opinions. The knowledge based system uses the knowledge encoded in some form such as rule-based systems, and decision tree. Generally, the construction of a failure effect knowledge base has been carried out by interviewing experts in failure effect and painstakingly translating the experts’ opinions into an appropriately structured set of rules (e.g., if-then). In Ref. 14, using min-max function and linguistic variables, authors proposed an approach that comprises the expert knowledge base rule and the failure factor interfaces to perform FEA with uncertain and imprecise information. This approach maps the knowledge obtained from experts into one or more if-then rule(s). However, the following sources of potential inconsistency may result in conflicting conclusions in the knowledge base system:

— Conflict of rules (i.e., ‘if’ parts of the rules are similar and ‘then’ parts are different)
— Subsumption (i.e., two or more rules have the same result, but one contains additional restriction on the situations in which it will succeed).

In addition, due to time consuming, complexity of consistency check, and difficulty of maintenance of knowledge base approach, the fuzzy TOPSIS approach for FEA was proposed to avoid the definition of a knowledge base supported by several qualitative rules. Though TOPSIS method has some advantages, it suffers from sensitivity analysis because the criterion with the highest score has disproportionate influence in the failure ranking process. On the other hand, in Ref. 17, to dilute conflict in decision group, authors presented a conflict resolution model to integrate multiple possibility distribution that can be used in Group-based Failure Effects Analysis (GFEA). The drawback of this model is the use of MINMAX function in aggregation technique, which is not adequate to study the failure effects and failure risk priority when tradeoffs exist among them. Therefore, using fuzzy aggregation and fuzzy compensation technique in MINMAX method, our objective is to present the GFEA approach in order to mitigate the problems (e.g., sensitivity and consistency analysis) associated with proposed FEA approaches (see basic concept of group decision making in Appendix-B).

2. Problem Description

In standard FMEA, either Risk Priority Number (RPN) or Risk Priority Category (RPC) is used not only to construct the system failure effects model (deterministic or fuzzy) but also to implement risk analysis. Most past publications used failure factors (Failure probability, Non-detection probability, Severity of effect, and
Expected cost) to assess either RPN or RPC of the potential failure. The RPN-based risk analysis, however, suffers from RPN shortcomings as follows\textsuperscript{6,9}:

(i) RPN does not take into account the quality of product.
(ii) Multiplication of factors in RPN expression is not always appropriate way for risk priority analysis.
(iii) The use of RPN does not satisfy the requirements of measurement.
(iv) Non-consistency of relation for different factors in RPN (e.g., linear relation between failure probability and probability scale and non-linear relation between non-detection probability and probability scale).
(v) Evolvement of identical RPN value for different sets of factors.
(vi) Overlooking of relative importance among failure factors.

Furthermore, traditional RPN cannot deal with

(vii) Tradeoffs among failure factors.
(viii) Divergence of experts’ opinion about failure factors or their importance.
(ix) Imprecise algebraic rule to assign a score to failure factors or their importance.

On the other hand, fuzzy approaches replace RPN with RPC that assigns a risk priority class to each cause of the failure in FEA. Because FEA is a group-based risk analysis technique (i.e., multi-based experts’ opinion),\textsuperscript{13} some researchers have used either decision techniques or knowledge base system to develop the group-based FEA model to deal with following issues:

(i) The opinions differ because the experts do not often agree on importance of the risk criteria (Failure factors).
(ii) Tradeoffs exist among risk criteria.
(iii) Experts are unable to assign an exact numerical value to importance of the risk criteria.
(iv) An expert is unable to express his/her opinion via numerical value for the relative relation of the potential failures to the certain risk criterion.

Therefore, due to human brain process, the use of fuzzy linguistic values is the most realistic method.\textsuperscript{21} Moreover, having more than one risk criterion requires a structured approach for GFEA problems. The knowledge base approach suffers from conclusion conflict due to conflict of rules and subsumption. In addition, it is a time consuming method, which is inflexible to change. On the other hand, setbacks of the decision-making approaches are the use of Maximin function and difficulty in performing sensitivity analysis. The new method must encompass the divergence of expert opinions that arises from fuzziness, and the complexity of both the failure effect and the importance of the functions/subsystems. Thus, the method must replace averaging operators (e.g., arithmetic mean, etc.) with compensatory operators to improve the accuracy of the aggregating process in fuzzy GFEA.\textsuperscript{18,19}
To develop the model, we consider a system that includes ‘I’ failures, and ‘J’ risk criteria under study by ‘K’ experts and uses the following notations:

**Notations**

- **i**: Index for failure, \( i = 1, 2, \ldots, I \)
- **j**: Index for risk criterion, \( j = 1, 2, \ldots, J \)
- **\( f_j \)**: ‘\( j \)’th risk criterion
- **k**: Expert index in group \( k = 1, 2, \ldots, K \)
- **\( \Psi \)**: Maximum number of categories
- **S**: A set of fuzzy linguistic variables (relative importance of decision factor or relative failure impact)
- **\( \xi \)**: Index for fuzzy variable \( \xi = 1, 2, \ldots, \Psi \)
- **\( \varsigma \)**: Index for fuzzy variable \( \varsigma = 1, 2, \ldots, \Psi \)
- **\( V_{ijk} \)**: Index of RPC factor for ‘\( i \)’th failure, ‘\( j \)’th risk criterion, and ‘\( k \)’th expert
- **\( S_\xi \)**: Fuzzy linguistic variable for defining the relative failure impact to risk criteria or the relative importance of the functions/subsystems.
- **\( I_k(f_j) \)**: The relative importance of ‘\( j \)’th criterion according to ‘\( k \)’th expert
- **\( P_{ik}(f_j) \)**: The relative level of impact of ‘\( i \)’th failure to ‘\( j \)’th criterion obtained from ‘\( k \)’th expert
- **RPC_{ik}** (RPC): Compensated Risk Priority Category (RPC) of the relative impact of the ‘\( i \)’th failure to the system according to ‘\( k \)’th expert’s opinion
- **RPC_{[ik]}** (RPC): ‘\( k \)’th maximum compensated Risk Priority Category of the relative impact of the failure ‘\( i \)’ to the system
- **\( \alpha_{jk} \)**: Index of importance category of ‘\( j \)’th risk criterion which is assigned by ‘\( k \)’th expert
- **\( \beta_{ijk} \)**: Index of ‘impact category of ‘\( i \)’th failure to ‘\( j \)’th risk criterion, which is assigned by ‘\( k \)’th expert
- **\( \gamma \)**: Degree of aggregation
- **\( \text{Int}[n] \)**: Integer value of a number, ‘\( n \)’
- **\( \text{Rnd}[n] \)**: Nearest integer value to a number, ‘\( n \)’
- **Set_k(i)**: A set composed of RPC’s of ‘\( i \)’th failure obtained from ‘\( K \)’ experts
- **Ord_k(i)**: A ranked set composed of RPC’s of ‘\( i \)’th failure obtained from ‘\( K \)’ experts

### 3. Group-based Failure Effects Analysis Model

Consider a system with ‘\( J \)’ risk criteria (i.e., failure factors and functions/subsystems). They can be expressed as score, percentage, and probability.
The risk criterion has its importance with respect to the perceptions of the member of a group (made up of ‘K’ experts). Also, ‘I’ potential failures are listed. Based on ‘k’th expert’s opinion, each one has a relative impact on the system relative to a risk criterion. As the experts cannot assign the exact number to either the importance of risk criteria or the failure impact on a system, the use of linguistic variables becomes the best alternative.

Table 2 presents nine linguistic variables denoted by ‘S’ with subscripted index rank to express the importance of the risk criteria and the system failure impacts relative to a risk criterion. To minimize the classification error and have a consistent GFEA model, we have based our linguistic variables upon the fuzzy method and Gray relational analysis used in Ref. 9. Gray theory provides a measure to analyze relationships between discrete quantitative and qualitative series.

Using the linguistic variables, the Decision Matrices are defined according to the experts’ opinion. For example, decision matrix of ‘k’th expert is

\[
MAT(k) = \begin{pmatrix}
I_k(f_1) & I_k(f_2) & \cdots & I_k(f_J) \\
\left( P_{1k}(f_1), P_{1k}(f_2), \cdots, P_{1k}(f_J) \right) \\
\left( P_{2k}(f_1), P_{2k}(f_2), \cdots, P_{2k}(f_J) \right) \\
\left( P_{Ik}(f_1), P_{Ik}(f_2), \cdots, P_{Ik}(f_J) \right)
\end{pmatrix}_{I\times J}
\]

where \( I_k(f_j) \) states the importance of ‘j’th risk criterion and \( P_{ik}(f_j) \) is the relative impact of ‘i’th failure on the system in relation with ‘j’th risk criterion (‘f_j’). In the case of divergence of decision matrices, we have developed the GFEA model not only to dilute the divergence in the failure risk analysis and but also to prioritize the system failures. The method uses the principal concept of ranking, maximum, and minimum for the linguistic variables as follows:
Definition 1. Ranking. Using the linguistic variables listed in Table 2, experts determine the importance of the risk criteria (i.e., failure factors including failure probability, non-detection probability, severity of effect, expected cost, and functions/subsystems importance) and the system failure impacts in relation with a risk criterion. Using ‘S’ to denote the term linguistic variable, and subscripted with an associated index rank, the rank for the failure factors and the function attributes can be expressed. For example, for impact of a failure on a system in relation to severity factor considered to be ‘VeryHigh (VH)’, it is represented in the index by $S_6$.

To assign RPC to failures based on a failure factor, we simply compare their indices. If a failure factor of failure ‘A’, denoted by ‘$S_\xi$’, is greater than the failure factor of failure ‘B’ in a system, denoted by ‘$S_\varsigma$’, the rank of failure ‘A’ is higher than the rank of failure ‘B’. In the other words, we write ‘$S_\xi > S_\varsigma$’ as $\xi > \varsigma$.

For example, in Appendix-A, the rank of the failure probability (failure factor) of three failures (Analog Personality Module, RS232 Com1, Power Conditioning) are assigned Low(L), High(H), and Medium(M) by expert 1. As presented in Table 2, the corresponding indices to ‘S’ for ‘L’, ‘M’, ‘H’ are 3, 4, and 5, respectively. Thus, in the opinion of expert 1, and within the definition that $S_5 > S_4 > S_3$, the rank of RS232 Com1 failure is higher than the rank of Analog Personality Module failure and of Power Conditioning failure relative to only failure probability factor.

Definition 2. Maximum and minimum for two linguistic variables. Consider two system failures with assigned system impact ranks (linguistic variables) relative to a specified failure factor (e.g., Non-detection probability). To identify the failure with higher system impact relative to the failure factor, a maximum function is required. With respect to Definition 1, the failure with maximum impact in relation to the specified failure factor in the system is the one with the highest rank (i.e., greater index of ‘S’), thus,

$$\text{Max}(S_\xi, S_\varsigma) = S_\xi$$ (1)

where index $\xi$ is greater than index $\varsigma$.

For example, using Table 2 to map the indices of two linguistic variables ‘Perfect (P)’ and ‘VeryHigh (VH)’, we assign indices 7 and 6 to ‘S’ for ‘Perfect (P)’ and ‘VeryHigh (VH)’, respectively. Thus, we write

$$\text{Max}(\text{‘P’}, \text{‘VH’}) = \text{Max}(S_7, S_6)$$

With respect to Definition 1, the rank of the linguistic variable ‘Perfect (P)’ is higher than that of ‘VeryHigh (VH)’ because $S_7 > S_6$. Thus,

$$\text{Max}(\text{‘P’}, \text{‘VH’}) = \text{‘P’}$$

Also, minimum of two fuzzy linguistic variables is equal to the lowest ranked one (i.e., smaller index of ‘S’), thus,

$$\text{Min}(S_\xi, S_\varsigma) = S_\varsigma$$ (2)

where index $\xi$ is greater than index $\varsigma$. 
To better describe the steps of the Group-based Failure Effects Analysis method, consider the decision matrix in Appendix A. For the example presented in Sec. 4, the new format of decision matrix combines all ‘K’ experts’ decision matrices. The first top three rows of the matrix show the relative importance of the risk criteria according to three experts. For example, experts 1, 2, and 3 expressed the importance of the Main Console VH, H, and P, respectively. The fourth row is the risk criteria made up of thirteen factors (i.e., Suspension tester subsystems and failure factors). The other rows are the potential failures and their relative impact to the risk criteria in the format of “X₁-X₂-X₃” corresponding to experts 1, 2, and 3. For example, the relative impact of the R232 interface disconnection (Failure) on the Main Console (Risk criterion) is categorized “H-VH-VH” by experts (i.e., expert1 assigns High(H) and expert2, and 3 assign VeryHigh(VH) to the relative impact of the R232(com) interface disconnection on the Main Console).

To dilute the divergence of experts’ opinion and to aggregate the opinions, the Group-based Failure Effects Analysis method has constituted the steps as follows:

**Step 0.** Determine a set of risk criteria and a set of potential failures.

In Appendix A, Table 6 shows Main Console, Main Console Motor, as effect criteria and fifty four potential failures in column one.

**Step 1.** Define importance of risk criteria.

Using linguistic variables of Table 2, ‘k’th expert determines the importance of all ‘J’ risk criterion (i.e., \(I_k(f_j)\), where \(k = 1, 2, \ldots, K\) and \(j = 1, 2, \ldots, J\). For example in Appendix A, Table 6, expert 1 expressed the importance of all failure factors (Main Console, Motor Control Console, ...) VH, P, VH, ..., respectively.

**Step 2.** Define the relative impact of the potential failure in relation with risk criteria.

Obtain the level of impact of the failure ‘i’ to the on the system in relation with the criterion ‘j’ from ‘k’th expert (e.g., \(P_{ik}(f_j)\)). For example, the relative impact of Circuit Breaker input cable (Failure) on the system in relation with the Main Console (Risk criterion) is categorized “N-H-H” by experts (i.e., expert1 assigns None(N) and expert2, and 3 assign High(H)).

**Step 3.** Determine risk priority category of failure ‘i’ according to perception of ‘k’th expert.

There are always tradeoffs among decision factors. The tradeoffs cause the identical RPC for different scenarios. In such situation, the use of the Minimum and Maximum functions is not adequate for studying failure risk analysis.\(^{17,18}\) Thus, to compute the category of the impact of failure according to certain expert’s opinion, Eq. (3) is used. It encompasses the divergence of expert’s opinion in decision
matrices. Also, to deal with tradeoffs among the decision factors, it is equipped to compensate operations proposed by Refs. 17, 18.

\[
RPC_{ik} = \min \{ \text{Cmp}(I_k(f_1), P_{ik}(f_1)), \text{Cmp}(I_k(f_2), P_{ik}(f_2)), \ldots, \text{Cmp}(I_k(f_J), P_{ik}(f_J)) \}
\]

(3)

where \( \text{Cmp}(I_k(f_j), P_{ik}(f_j)) \) is the compensated maximum function for aggregating the importance of \( j \)’th risk criterion (i.e., \( I_k(f_j) \)) with \( i \)’th failure effect on the system in relation with \( j \)’th risk criterion (i.e., \( P_{ik}(f_j) \)) according to \( k \)’th expert’s opinion. By substituting \( I_k(f_j) \) and \( P_{ik}(f_j) \) with corresponding indices according to Table 2 (i.e., \( S_{\alpha jk} \) and \( S_{\beta ijk} \)) in Eq. (3), we get

\[
RPC_{ik} = \min \{ \text{Cmp}(S_{\alpha 1k}, S_{\beta i1k}), \text{Cmp}(S_{\alpha 2k}, S_{\beta i2k}), \ldots, \text{Cmp}(S_{\alpha Jk}, S_{\beta ijk}) \}
\]

(4)

where

\[
\text{Cmp}(S_{\alpha jk}, S_{\beta ijk}) = S_{V_{ijk}}
\]

(5)

and

\[
V_{ijk} = \text{Rnd} \left( \gamma \cdot \max(\alpha_{jk}, \beta_{ijk}) + (1 - \gamma) \cdot \text{Int} \left( \frac{\alpha_{jk} + \beta_{ijk}}{2} \right) \right)
\]

(6)

where \( \gamma \) is the degree of aggregation (0 ≤ \( \gamma \) ≤ 1). For \( \gamma \) equal to one, Eq. (5) is same as Eq. (1) and setting \( \gamma \) equal to zero yields the arithmetic mean for Eq. (5). Thus, by substituting Eqs. (5) and (6) into Eq. (4), we have \( RPC_{ik} \) that indicates risk priority category of \( i \)’th failure in the system based on \( k \)’th expert’s perception. For example, the \( RPC \) of failure 1 in Appendix A (i.e., RS232(Com1)) based on expert 1 is expressed by

\[
RPC_{11} = \min \{ \text{Cmp}(H,VH), \text{Cmp}(N,P), \text{Cmp}(N,VH), \text{Cmp}(N,VH), \text{Cmp}(N,H), \text{Cmp}(N,M), \text{Cmp}(N,VH), \text{Cmp}(N,VH), \text{Cmp}(N,VH), \text{Cmp}(N,VH), \text{Cmp}(H,VH), \text{Cmp}(H,VH), \text{Cmp}(H,VH) \}
\]

Using indices of Table 2, \( RPC_{11} \) is defined as

\[
RPC_{11} = \min \{ \text{Cmp}(S_5, S_6), \text{Cmp}(S_1, S_7), \text{Cmp}(S_1, S_6), \text{Cmp}(S_1, S_6), \text{Cmp}(S_1, S_5), \text{Cmp}(S_1, S_4), \text{Cmp}(S_1, S_5), \text{Cmp}(S_1, S_6), \text{Cmp}(S_1, S_6), \text{Cmp}(S_5, S_6), \text{Cmp}(S_5, S_6), \text{Cmp}(S_5, S_5) \}
\]

With respect to Eqs. (5), and (6) and Definition 1 and \( \gamma = 0.8 \), we get

\[
RPC_{11} = \min \{ S_6, S_5, S_4 \} = S_4.
\]

The above means that the risk priority category of the failure 1 is “Medium” according to expert 1. Similarly, \( RPC \) of Failure 1 is defined “Low” for experts 2 and 3.
Perform steps 1 to 3 for all experts to determine $\text{RPC}_{ik}$ where $i = 1, 2, \ldots, I$, and $k = 1, 2, \ldots, K$.

**Step 4.** Rank the RPC’s of the failure.

Using the Definition 1, we generate a set of ranked risk priority category for each failure. Thus, the set of RPC’s for ‘i’th failure is expressed by

$$\text{Set}_K(i) = \{\text{RPC}_{i1}, \text{RPC}_{i2}, \ldots, \text{RPC}_{iK}\}$$  \hspace{1cm} (7)

with respect to Definition 1, the ranked set is

$$\text{Ord}_K(i) = \{\text{RPC}_{[i1]}, \text{RPC}_{[i2]}, \ldots, \text{RPC}_{[iK]}\}$$  \hspace{1cm} (8)

where, $\text{RPC}_{[ik]}$ is ‘$k$’th maximum in the ranked list of the set element.

For example, the set of RPC’s for failure 1 in Appendix A based on three experts is

$$\text{Set}_3(1) = \{M, L, L\}.$$  

Thus, the ranked set is

$$\text{Ord}_3(1) = \{M, L, L\}.$$

**Step 5.** Determine aggregation categories.

Depending upon number of experts agreed on certain level on failure impact, the aggregation category must be generated. Assume ‘$\varepsilon$’ is total number of experts agreed on certain failure and criterion. Using ratio $\varepsilon/K$, the aggregation category maps ‘$\varepsilon$’ to relative linguistic set of Table 2. For example, the aggregation category is ‘Perfect’ when all experts agree on certain level of the failure effect on the system (i.e., $\varepsilon = K$). To map ‘$\varepsilon$’, the Eq. (9) presents the aggregation function used in the model.

$$\text{Agg}(\varepsilon) = S_{\text{Int}[1 + (\varepsilon \cdot \Psi - 1)/K]}$$  \hspace{1cm} (9)

where ‘$\varepsilon$’ is the number of experts that satisfies aggregation function ($0 \leq \varepsilon \leq K$).

Using Table 2, Table 3 presents the aggregation categories of the example given in Appendix A for a group comprises three experts ($K = 3$) and seven levels of linguistic variables ($\Psi = 7$). As a result, the level of agreement will be increased as shown in Table 3 by increasing ‘$\varepsilon$’.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\Psi$</th>
<th>$\text{Int}[1 + (\varepsilon \cdot \Psi - 1)/K]$</th>
<th>Index</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>1</td>
<td>$S_1$</td>
<td>Non (N)</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>3</td>
<td>$S_3$</td>
<td>Low (L)</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>5</td>
<td>$S_5$</td>
<td>High (H)</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>7</td>
<td>$S_7$</td>
<td>Perfect (P)</td>
</tr>
</tbody>
</table>
Step 6. Compute comprehensive risk priority category of the potential failures.

In the Group-based FEA, the failure is associated with a set that includes ordered elements (i.e., RPC’s) corresponding to the members of group. The comprehensive risk priority category for each failure is the aggregation of the RPC’s of the certain failure. Thus, by substituting \( \text{Agg}(\varepsilon) \) function (Eq. (9)) into Eq. (10), the comprehensive aggregated failure impact of ‘\( i \)’th failure in the system is expressed by

\[
\text{RPC}_i = \text{Max}\{\text{Min}(\text{Agg}(\varepsilon), \text{RPC}_{[\varepsilon]}), \text{Min}(\text{Agg}(\varepsilon - 1), \text{RPC}_{[\varepsilon-1]}), \ldots, \\
\text{Min}(\text{Agg}(1), \text{RPC}_{[1]})\}
\]  

(10)

where \( \text{RPC}_{[k]} \) is ‘\( k \)’th maximum in the ranked list of RPC of ‘\( i \)’th failure and \( i = 1, 2, \ldots, I \).

For example, using Table 3 and Ord\(_3\)(1) in step 4, the RPC\(_{[13]}\) is defined as

\[
\text{RPC}_1 = \text{Max}\{\text{Min}(L,M), \text{Min}(M,L), \text{Min}(P,L)\} = L
\]

By performing the step 6 for all failures \( (i = 1, 2, \ldots, I) \) and using Definition 1, a set made up of the ranked comprehensive risk priority category can be generated.

Step 7. Perform sensitivity analysis.

In order to ensure the consistency of GFEA result, a slight change in either the level of risk criteria importance \( I_k(f_j) \) or the level of failure impact on risk criteria \( P_{ik}(f_j) \) for \( I = 1, 2, \ldots, I, j = 1, 2, \ldots, J \) and \( k = 1, 2, \ldots, K \). If slightly change of them produces a completely different result, tune the aggregation degree to dilute the divergence of the certain expert on the result. Moreover, by changing \( \gamma \) from 0 to 1 with incremental step equal to 0.1 and perform steps 3 to 6, generate RPC’s corresponding to the certain value of \( \gamma \). Based on Pareto Chart, if the 15% of top ranked failures is always similar for different value of \( \gamma \), the GFEA result is consistent and independent from \( \gamma \). Otherwise, conflict level of expert’s opinion needs to be diluted by removing the predefined items of decision matrix \( (I_k(f_j), P_{ik}(f_j)) \) that slightly change them causes inconsistency of the result. Perform steps 3 to 6 to find the result. A solved example in the next section is presented to demonstrate sensitivity analysis procedure.

4. Example

Military vehicles are periodically required their suspension to be tested. Thus, a new model of suspension tester is developed to perform the test in the field without returning the vehicles to shop. This device comprises digital and analog circuitry and hydraulic subsystems as shown in Appendix A. The device performs the test in several steps (i.e., engage, lift, release and disengage). In order to study the RPC of the failures and rank them for improving physical design, a group including three experts (e.g., design engineer, reliability engineer, and sales/marketing engineer) is carrying out FEA. The system has 12 risk criteria (9 subsystems and 3 failure factors) as shown in the fourth row of the decision matrices in Appendix A. The
Table 4. GFEA result for $\gamma = 0.8$.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Failures</th>
<th>Failure effect/Severity</th>
<th>Rank</th>
<th>Failures</th>
<th>Failure effect/Severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120VAC-DPST Relay</td>
<td>High</td>
<td>3</td>
<td>3A Fuse-24 VDC</td>
<td>Low</td>
</tr>
<tr>
<td>1</td>
<td>120VAC-Circuit Breaker</td>
<td>High</td>
<td>3</td>
<td>3A Fuse- power supply</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Circuit Breaker input Cable</td>
<td>Medium</td>
<td>3</td>
<td>24VDC power supply</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Circuit Breaker input SW 1</td>
<td>Medium</td>
<td>3</td>
<td>LoadPad/Wheel Acc./Hull Acc./Cable</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Circuit Breaker input SW 2</td>
<td>Medium</td>
<td>3</td>
<td>MV10-Signal Conditioning Board</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Circuit Breaker input SW 3</td>
<td>Medium</td>
<td>3</td>
<td>Power conditioning</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Acc. 1A (-15)V</td>
<td>Medium</td>
<td>3</td>
<td>1A Fuse-dual power supply-15 VDC</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Acc. 1A (+15)V</td>
<td>Medium</td>
<td>3</td>
<td>1A Fuse- input power supply</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>APM (Analog Personality Module)</td>
<td>Medium</td>
<td>3</td>
<td>ICS 3150-020</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>120VAC-Receptable Connection</td>
<td>Medium</td>
<td>3</td>
<td>Encoder</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Winsys-RTD DM6856</td>
<td>Medium</td>
<td>3</td>
<td>120VAC-DMC Fuse</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Winsys-RTD DM6430</td>
<td>Medium</td>
<td>3</td>
<td>Magnetic position switch on Rig</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Digital Circuit Breaker- Cable</td>
<td>Medium</td>
<td>3</td>
<td>DMC 1412</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Digital Circuit Breaker- Switch 1</td>
<td>Medium</td>
<td>3</td>
<td>Servo Amp</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Digital Circuit Breaker- Switch 2</td>
<td>Medium</td>
<td>3</td>
<td>ICM 1460</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Digital Circuit Breaker- Switch 3</td>
<td>Medium</td>
<td>3</td>
<td>Com1</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Cylinder Leakage</td>
<td>Medium</td>
<td>3</td>
<td>RTD SS8</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>RAM Excessive play</td>
<td>Medium</td>
<td>3</td>
<td>Keypad</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Seized Cylinder</td>
<td>Medium</td>
<td>3</td>
<td>Winsys-DIO</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Pressure Relief Valve leakage</td>
<td>Medium</td>
<td>3</td>
<td>Winsys Fuse</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Pressure Relief Valve open</td>
<td>Medium</td>
<td>3</td>
<td>Winsys-120 VCA power</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Solenoid</td>
<td>Medium</td>
<td>3</td>
<td>Electromotor</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Valves</td>
<td>Medium</td>
<td>3</td>
<td>Pump</td>
<td>Low</td>
</tr>
<tr>
<td>3</td>
<td>RS232(Com1) interface disconnected</td>
<td>Low</td>
<td>3</td>
<td>Reservoir</td>
<td>Low</td>
</tr>
<tr>
<td>3</td>
<td>Acc. Cable</td>
<td>Low</td>
<td>3</td>
<td>Circuit Breaker Output-3A Fuse</td>
<td>Low</td>
</tr>
<tr>
<td>3</td>
<td>Acc.1A fuse</td>
<td>Low</td>
<td>3</td>
<td>Circuit Breaker Output-3A fuse(2)</td>
<td>Low</td>
</tr>
<tr>
<td>3</td>
<td>Acc. Dual power supply</td>
<td>Low</td>
<td>3</td>
<td>Circuit Breaker Output-24VDC</td>
<td>Low</td>
</tr>
<tr>
<td>3</td>
<td>AFF Borad</td>
<td>Low</td>
<td>3</td>
<td>Circuit Breaker Output-Cable</td>
<td>Low</td>
</tr>
</tbody>
</table>
Rows 1 to 3 of the decision matrices are the relative importance (category) of those risk criteria corresponding to experts 1, 2, and 3. Moreover, the experts identified 59 potential failures that were listed in row 5 to the end. In each row, a failure is associated with category of its impact to defined criteria in format of “$X_1$-$X_2$-$X_3$” corresponding to experts 1, 2, and 3. For example, the relative impact of the R232(com) interface disconnection (Failure) on the Main Console (Risk criterion) is defined “H-VH-VH” by experts (i.e., expert 1 assigns High(H) and experts 2, and 3 assign Very High(VH) to the relative impact of the R232(com) interface disconnection on the Main Console). Table 6 (see Appendix A) depicts divergence/conflict of experts’ opinion that comes from human brain process and the fuzziness of the failure impact. Shortcomings of single-based expert methods that are RPN-based or RCP-based, risk analysis does not allow using them for such system. Therefore, using GFEA model with a degree of aggregation (e.g., equal to 0.8), Table 4 shows subsystems such as Winsys, power supply, and circuit breaker have high ranked failures and they need to be reengineered. To ensure the robustness of the model, the sensitivity analysis is performed. Table 5 presents that the 15% top ranked failures are the same for different values of the aggregation degree and ranking differentiation for $\gamma > 0.8$ is justifiable than ranking differentiation for $\gamma < 0.8$. RPC of the failures is different and it corresponds to the value of $\gamma$ because $\gamma$ compensates the aggregation function with arithmetic mean. Furthermore, a slight change of parameters in decision matrices does not result in completely different RPC’s, that lead to the consistency of GFEA method.

5. Conclusion

The FEA methods focus on decision leading to improve the reliability, quality, and safety of the system. They are classified to deterministic and fuzzy approaches, which are single-expert based or multi-expert based. The methods (i.e., knowledge base system, decision support) use RPN or RPC to evaluate the risk level of the system. However, there are sources (i.e., Conflict of rules, Subsumption) of potential inconsistency may result in conflicting conclusions in the knowledge base system. On the other hand, the decision support methods do not take into account the
tradeoffs among the risk factors and are unable to perform sensitivity analysis on the failure factors.

This paper presented a group-based FEA model to aggregate the intuitive decision of experts about importance of failure factors and failure effect on these factors. As demonstrated, the approach is built upon not only aggregating the perceptions of experts concerning the importance of failure factor and failure effect, but also the ranking of these failures with respect to their RPC. Furthermore, using a degree of aggregation ($\gamma$) and a slight change in parameters, the newly developed model can perform sensitivity analysis to judge the consistency of the result. By having consistent ranked failures as a road map, the reengineering process can be executed for improving the functions/subsystems that either generate the top ranked failures, or are affected by the top ranked failures. For future work, this approach requires the detection/isolation/recovery method for finding the inconsistence experts and diluting their opinions in GFEA.

References


**About the Authors**

Dr. K. Jenab received B.Sc degree from Department of Industrial and System Engineering, Isfahan University of Technology in 1989, M.Sc degree from Department of Industrial Engineering, Tehran Polytechnic in 1992, and Ph.D degree from Department of Industrial Engineering, Iran University of Science and Technology in 2000. He served as Senior Engineer/Manager in Avionics, Auto, and High-tech industries for 16 years. He held academic appointment with department of Industrial and System Engineering in Azad University-Tehran. He joined National Research Council Canada as Senior Research Officer and participated in several international research projects. He published several papers in international scientific journals based on his experiences in Avionics, Transportation, and Telecommunication industries. Recently, Dr. Jenab provides consultant services (System Design, and Reliability, Quality, and Safety Engineering) to the Avionics, Transportation, and Telecommunication companies, such as Honeywell-ASC, Air Bus industry, Bombardier Aerospace, Messier-Dowty Inc, Alcatel Inc, National Research Council Canada, etc.

Dr. Dhillon has served as Chairman/Director/Acting Director of the Mechanical Engineering Department/Engineering Management Program for over 10 years. He has published over 300 articles and 28 books: Wiley (1981), Van Nostrand (1983), Marcel Dekker (1984), Pergamon (1986), etc. Many of his books are translated into languages such as German, Russian and Chinese and he is/has been on the editorial board of 7 international scientific journals. Dr. Dhillon is recipient of many awards/honors including Austin J. Bonis Award (American Society for Quality
306  K. Jenab & B. S. Dhillon Control, Merit Award (Society of Reliability Engineers), and Glinski Award (Faculty of Engineering). He is listed in many Who’s Who documents and has served as consultant to various organizations. Prof. Dhillon has lectured in over 50 countries including keynote addresses at various scientific conferences held in North America, Europe, Asia, and Africa.

Appendix A: Decision Matrix

Table 6. Decision Matrices.
Appendix B: Basic Concept of Group Decision Making

Within group decision-making processes that are based on fuzzy linguistic preference, there exists a set of alternatives (Potential Failure) and a set of decision-makers that are denoted by failure and expert sets, respectively. Decision-maker (DM) bases his/her decision upon certain risk criteria (i.e., failure factors and functions/subsystems importance). These criteria can be independent or dependent. When criteria are independent, the importance weights of criteria are treated as coefficients of an additive aggregation rule. By defining $X_{ijk}$ as fuzzy preference relation between ‘$i$’th alternative (the potential failures) and ‘$j$’th criterion (failure factors and functions/subsystems) taken from ‘$k$’th DM, the decision matrix for ‘$k$’th DM can be developed as shown in Fig. 1. What makes the decision process difficult is the involvement of both the competency of the alternatives and the divergence of decision makers’ perception of those alternatives. In such situations, not only is the matching technique used to find a match between alternatives and preferences for each individual decision-maker, but also aggregating techniques are implemented, as a second step to combine perceptions of all decision-makers.

![Fig. 1. Decision matrix of $DM_k$.](image-url)